

# Thermodynamics of oxygen defective $\text{TiO}_{2-x}$ : The Magneli phases.

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# Magneli Phases

Figure 1a

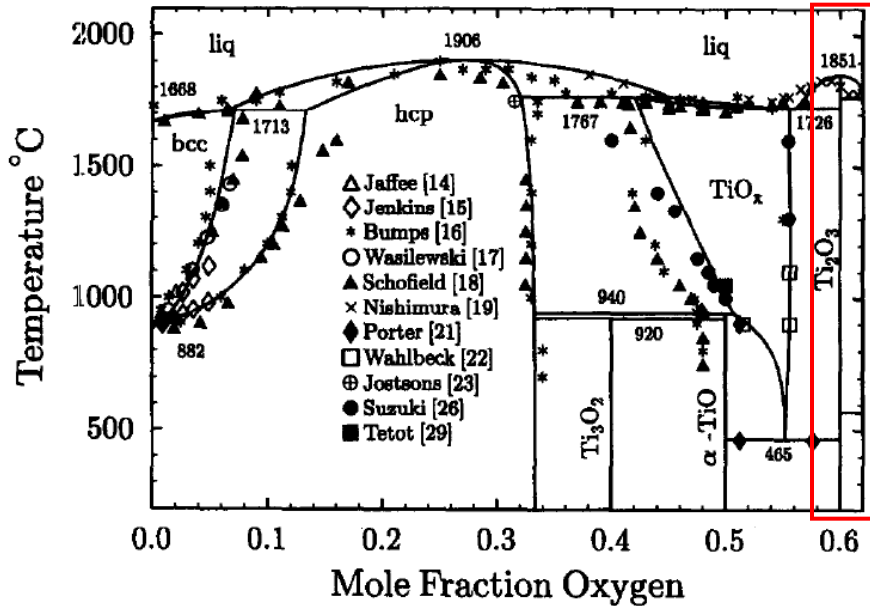
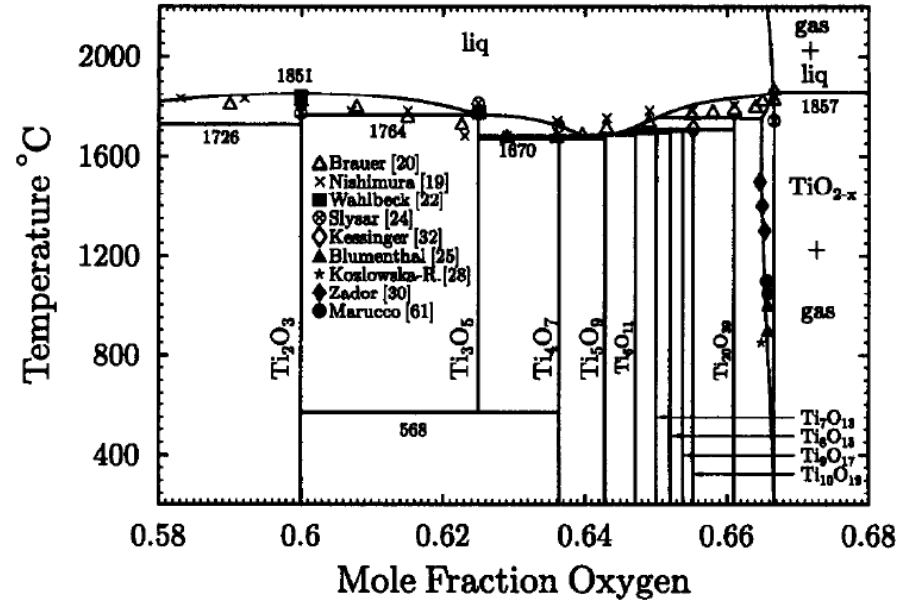


Figure 1b



$T_nO_{2n-1}$  composition,  $4 \leq n \leq 9$ . Oxygen defects in {121} planes.

$Ti_4O_7$  at  $T < 154K$  insulator with 0.29eV band gap<sup>(1)</sup>.

$T_4O_7$  Metal-insulator transition at 154K, with sharp decrease of the magnetic susceptibility.

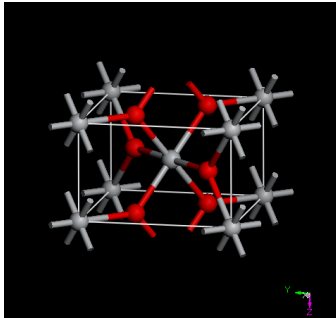
(1) K. Kobayashi *et al.*, Europhysics Lett., Vol. 59, pp. 868-874, 2002.

(2) W. Masayuki *et al.*, J. of Luminiscence, Vol. 122-123, pp. 393-395, 2007.

(3) P. Waldner and G. Eriksson, Calphad Vol. 23, No. 2, pp. 189-218, 1999.

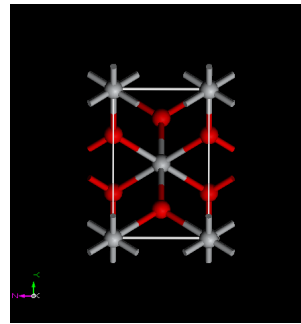
# Magneli Phases: $T_4O_7$ crystalline structure

Figure 3a



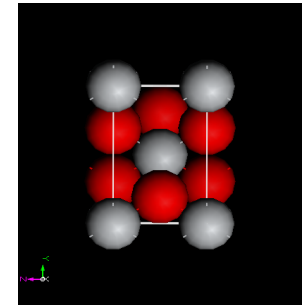
Rutile unit cell

Figure 3b



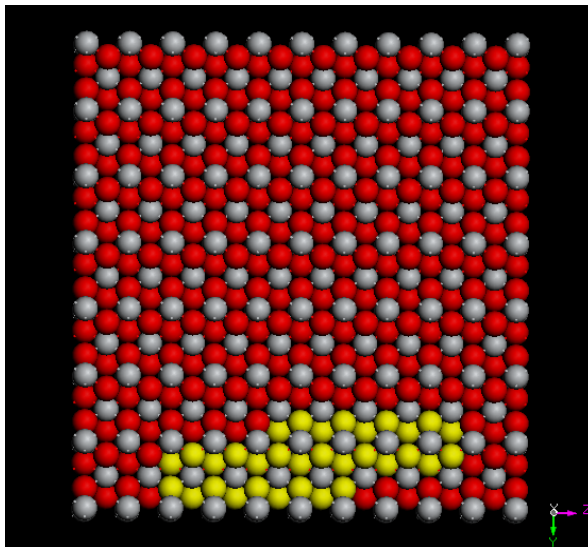
View along the a  
lattice parameter

Figure 3c



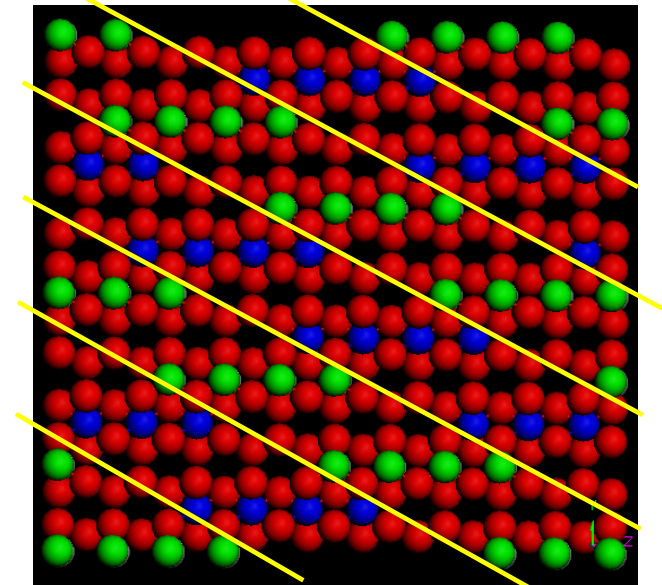
View of Hexagonal  
oxygen arrangement

Figure 3d



View of Hexagonal  
oxygen network

Figure 3e



Metal nets in antiphase.  $(121)_r$   
Crystallographic shear plane.

# Technical details of the calculations

## CASTEP

Local density functional: LDA

Ultrasoft pseudopotentials replacing core electrons

Plane waves code

Supercell approach

Simulated systems: Oxygen point-defective supercell, Magneli phases supercells, Titanium bulk metal.

## CRYSTAL

Hybrid density functional: B3LYP,  
GGA Exchange  
GGA Correlation  
20% Exact Exchange

All electron code. No pseudopotentials

Local basis functions: atom centred Gaussian type functions.

Ti: 27 atomic orbitals, O: 18 atomic orbitals

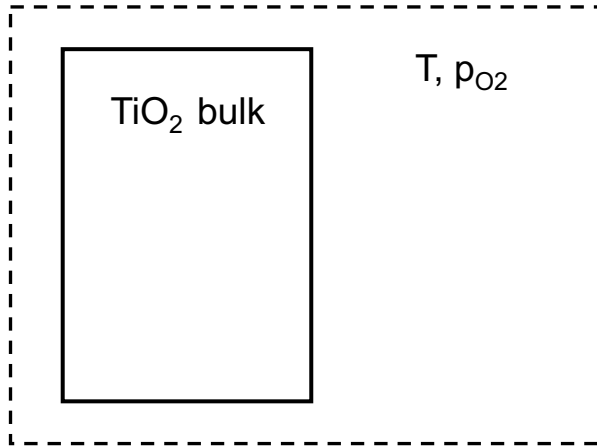
Supercell approach

Simulated systems: Oxygen point-defective supercell, Magneli phases supercells, Oxygen molecule.

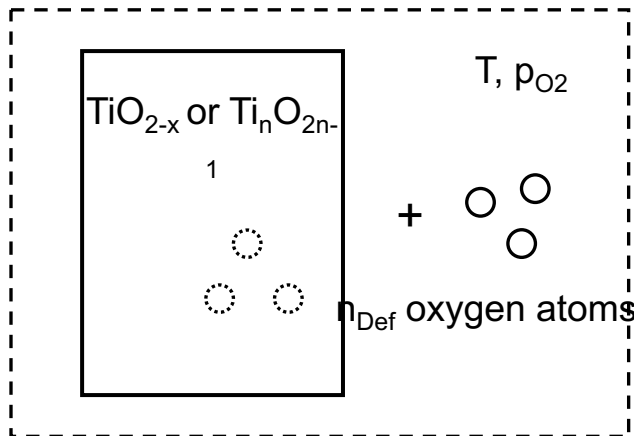
# Defect Formation Energies: Thermodynamical Formalism

Figure 5a

Initial state



Final state



$$\Delta G_f^{Def}(T, p_{O_2}) = \frac{1}{n_{TiO_2}} \left( G^{supcell}(T, p_{O_2}) + n_O^{Def} \mu_O^{ref}(T, p_{O_2}) \right) - \frac{1}{n_{TiO_2}} \left( n_{TiO_2} \mu_{TiO_2}^{bulk}(T, p_{O_2}) \right) \quad (1)$$

$$\Delta G_f^{Def}(T, p_{O_2}) = \frac{1}{n_{TiO_2}} \left( E^{supcell}(0K) - n_{TiO_2} E_{TiO_2}^{bulk}(0K) \right) +$$

Phonon contribution

$$+ \frac{1}{n_{TiO_2}} \left( F_{Vib}^{supcell}(T) - n_{TiO_2} F_{Vib}^{bulk}(T) \right) +$$

pV contribution

$$+ \frac{1}{n_{TiO_2}} p_{O_2} \left( V_{Vib}^{Supcell} - n_{TiO_2} V_{TiO_2}^{bulk} \right) +$$

$$+ \frac{n_O^{Def}}{n_{TiO_2}} \mu_{O_2}^{ref}(T, p_{O_2}) \quad (2)$$

$$\Delta G_f^{Def}(T, p_{O_2}) = \frac{1}{n_{TiO_2}} \left( E^{supcell}(0K) - n_{TiO_2} E_{TiO_2}^{bulk}(0K) \right) +$$

$$+ \frac{n_O^{Def}}{n_{TiO_2}} \mu_O^{ref}(T, p_{O_2}) \quad (3)$$

# Defect Formation Energies: Oxygen chemical potential

$$\Delta G_f^{Def}(T, p_{O_2}) = \frac{1}{n_{TiO_2}} \left( E^{supcell}(0K) - n_{TiO_2} E_{TiO_2}^{bulk}(0K) \right) + \frac{n_O^{Def}}{n_{TiO_2}} \mu_O^{ref}(T, p_{O_2}) \quad (3)$$

Limits for the oxygen chemical potential:

$$E_0 + \frac{\Delta G_f^{Rutile}(T^0, p_{O_2}^0)}{2} \leq \mu_O(p_{O_2}, T) \leq E_0 \quad (4)$$

Hard limit
Soft limit

Assuming the oxygen behaves as an ideal gas:

$$\mu_{O_2}(p_{O_2}, T) = 2\mu_O(p_{O_2}, T) = E_0 + (\mu_{O_2}^0 - E_0) \frac{T}{T^0} - \frac{5k}{2} T \ln\left(\frac{T}{T^0}\right) + kT \ln\left(\frac{P_{O_2}}{P_{O_2}^0}\right) \quad (5)$$

Oxygen molecule's total energy at 0K

Oxygen molecule's standard chemical potential at T=298K and p<sub>O2</sub>=1atm

Expression (5) allows the calculation of μ<sub>O2</sub><sup>0</sup>(T, p<sub>O2</sub>) at any T and p<sub>O2</sub>

# Oxygen chemical potential

## CASTEP

$$\mu_{O_2}^0(p^0, T^0) = \frac{2}{y} \left( \mu_{M_x O_y}^{bulk} - x \mu_M^{bulk} - \Delta G_{M_x O_y}^0(p^0, T^0) \right)$$

$M_x O_y$ : ZnO, Anatase, Rutile,  $Ti_4 O_7$ ,  $Ti_3 O_5$

$$\mu_{O_2}^0(T^0, p_{O_2}^0) = \mu_{mean} \pm \Delta \mu$$

Now  $E_0$  has to be calculated

## CRYSTAL

$E_0$  and the 0K total energy of the oxygen atom are calculated with CRYSTAL.

	Exp.	PW-GGA (4)	CRYSTAL
Binding energy [eV]	2.56	3.6	2.53
Bond length [ang]	1.21	1.22	1.23

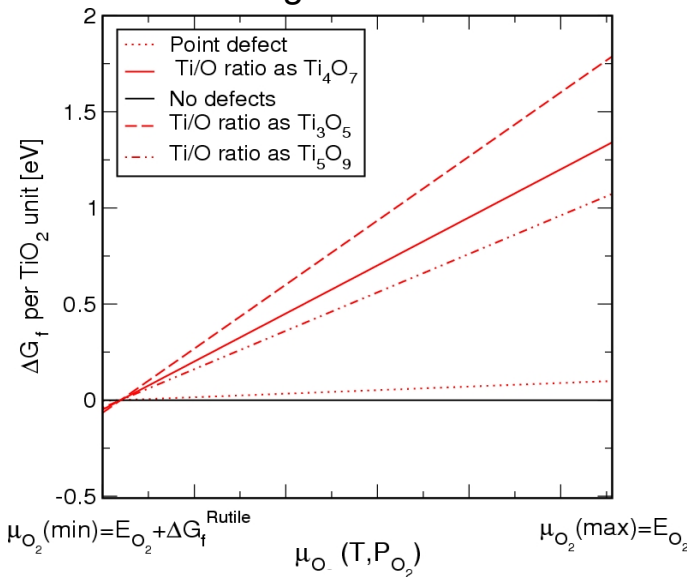
Now  $\mu_{O_2}^0$  has to be calculated

$$\mu_{O_2}(p_{O_2}, T) = A(T - T \ln(T)) - \frac{1}{2} B T^2 - \frac{1}{6} C T^3 - \frac{1}{12} D T^4 - \frac{E}{2T} + F - G T \quad (6) \quad \begin{matrix} T > 298K \text{ and} \\ p_{O_2} = 1 \text{atm} \end{matrix}$$

$$\mu_{O_2}(p_{O_2}, T) = E_0 + (\mu_{O_2}^0 - E_0) \frac{T}{T^0} - \frac{5k}{2} T \ln\left(\frac{T}{T^0}\right) + kT \ln\left(\frac{P_{O_2}}{P_{O_2}^0}\right) \quad (5) \quad T > 0K \text{ and any } p_{O_2}$$

# Results for the Magneli phases

Figure 8a



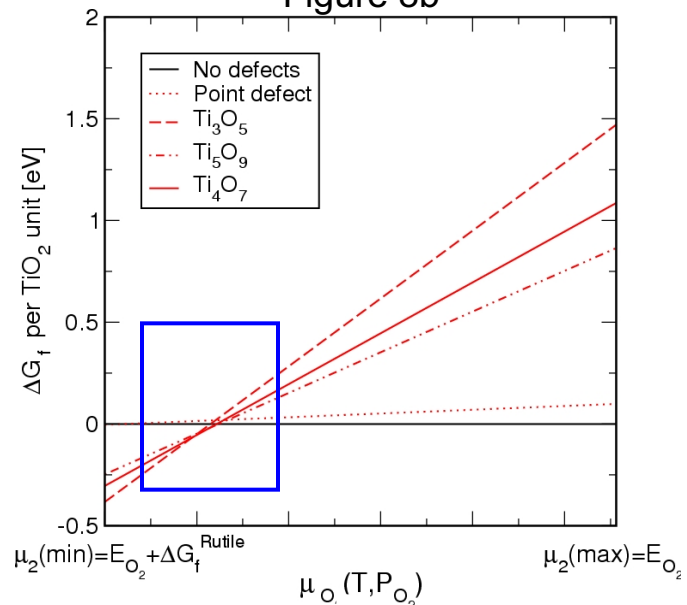
Isolated defects

$$\Delta G_{f \text{ isolat.}}^{Def} (T, p_{O_2}) = \frac{n_O^{Def}}{n_{TiO_2}} \left( E^{\text{supcell}} (0K) - n_{TiO_2} E_{TiO_2}^{bulk} (0K) + \mu_O^{ref} (T, p_{O_2}) \right)$$

$$\left( \frac{n_O^{Def}}{n_{TiO_2}} \right)_{Ti_4O_7} = \frac{1}{4}$$

$$\Delta G_{\left( \frac{Ti}{O} \right) \text{ like } Ti_4O_7}^{Def} (T, p_{O_2}) = \frac{1}{4} \left( E^{\text{Supcell}} (0K) - 27 E_{TiO_2}^{bulk} (0K) + \mu_O^{ref} (T, p_{O_2}) \right)$$

Figure 8b



Magneli phases

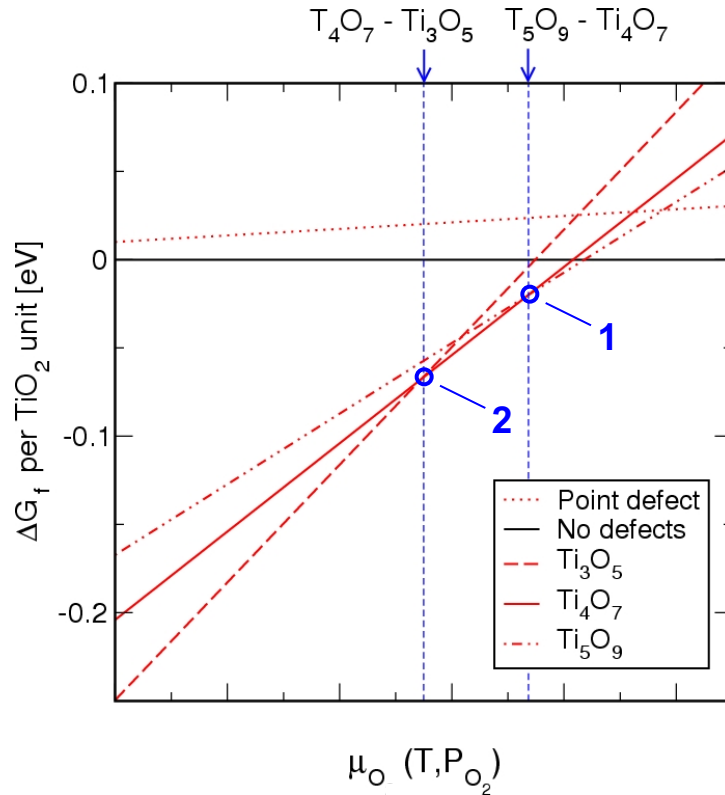
$$\Delta G_f^{Def} (T, p_{O_2}) = \frac{1}{n_{TiO_2}} \left( E^{\text{supcell}} (0K) - n_{TiO_2} E_{TiO_2}^{bulk} (0K) \right) + \frac{n_O^{Def}}{n_{TiO_2}} \mu_O^{ref} (T, p_{O_2})$$

$$\left( \frac{n_O^{Def}}{n_{TiO_2}} \right)_{Ti_4O_7} = \frac{1}{4}$$

$$\Delta G_{Ti_4O_7}^{Def} (T, p_{O_2}) = \frac{1}{16} \left( E^{Ti_4O_7} (0K) - 8 E_{TiO_2}^{bulk} (0K) \right) + \frac{1}{4} \mu_O^{ref} (T, p_{O_2})$$



# Results for the Magneli phases



$$\Delta G_{\text{Ti}_3\text{O}_5}^{\text{Def}}(\mu_{\text{O}}) = K_{\text{Ti}_3\text{O}_5} + \frac{\mu_{\text{O}}}{3}$$

$$\Delta G_{\text{Ti}_4\text{O}_7}^{\text{Def}}(\mu_{\text{O}}) = K_{\text{Ti}_4\text{O}_7} + \frac{\mu_{\text{O}}}{4}$$

$$\Delta G_{\text{Ti}_5\text{O}_9}^{\text{Def}}(\mu_{\text{O}}) = K_{\text{Ti}_5\text{O}_9} + \frac{\mu_{\text{O}}}{5}$$

Equilibrium point 1:

$$\Delta G_{\text{Ti}_4\text{O}_7}^{\text{Def}}(\mu_{\text{O}}) = \Delta G_{\text{Ti}_5\text{O}_9}^{\text{Def}}(\mu_{\text{O}}) \Rightarrow \mu_{\text{Ti}_4\text{O}_7 - \text{Ti}_5\text{O}_9}^{\text{eq}}$$

Equilibrium point 2:

$$\Delta G_{\text{Ti}_4\text{O}_7}^{\text{Def}}(\mu_{\text{O}}) = \Delta G_{\text{Ti}_3\text{O}_5}^{\text{Def}}(\mu_{\text{O}}) \Rightarrow \mu_{\text{Ti}_4\text{O}_7 - \text{Ti}_3\text{O}_5}^{\text{eq}}$$



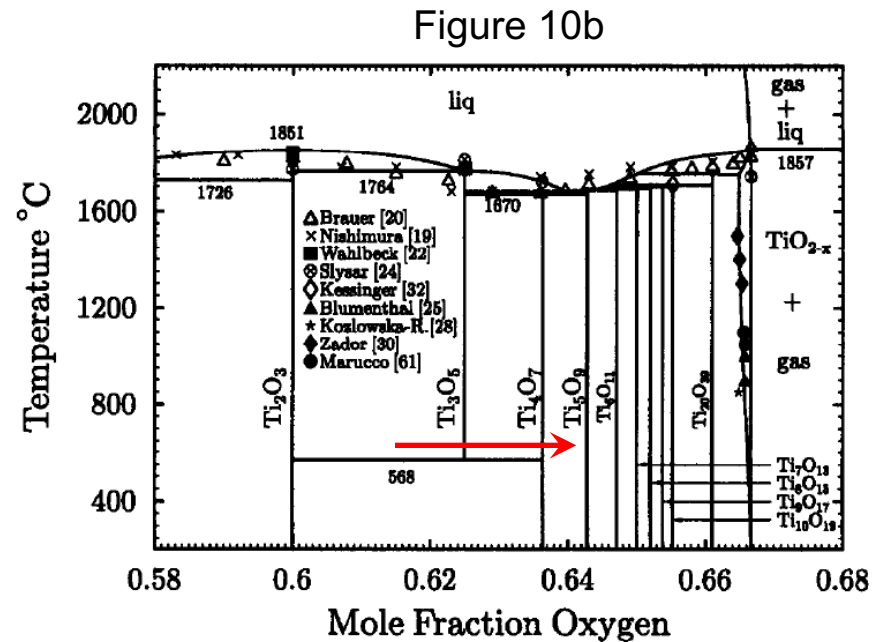
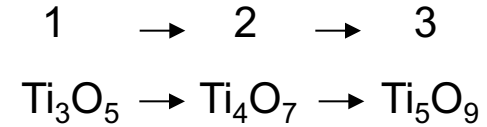
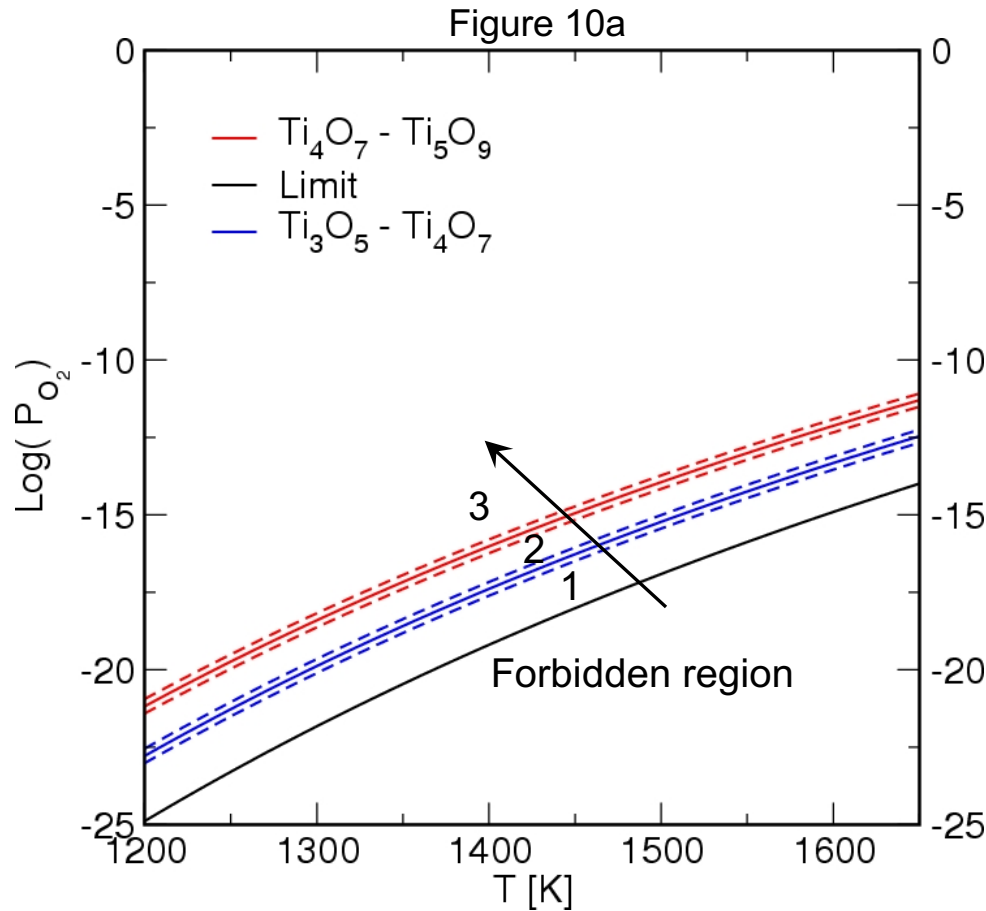
$$\mu_{\text{Ti}_4\text{O}_7 - \text{Ti}_5\text{O}_9}^{\text{eq}} = E_0 + (\mu_{\text{O}_2}^0 - E_0) \frac{T}{T^0} -$$

$$-\frac{5k}{2} T \ln\left(\frac{T}{T^0}\right) + kT \ln\left(\frac{P_{\text{O}_2}}{P_{\text{O}_2}^0}\right)$$



Relationship between  $p_{\text{O}_2}$  and T in the phase equilibrium.

# Results for the Magneli phases



$$\log_{10}(p_{\text{O}_2}) = \frac{K_{\text{Ti}_4\text{O}_7 - \text{Ti}_5\text{O}_9}^{\text{eq}}}{T} + K_1 \ln\left(\frac{T}{T^0}\right) + K_2$$

# CASTEP Results for the Magneli phases

Figure 10a

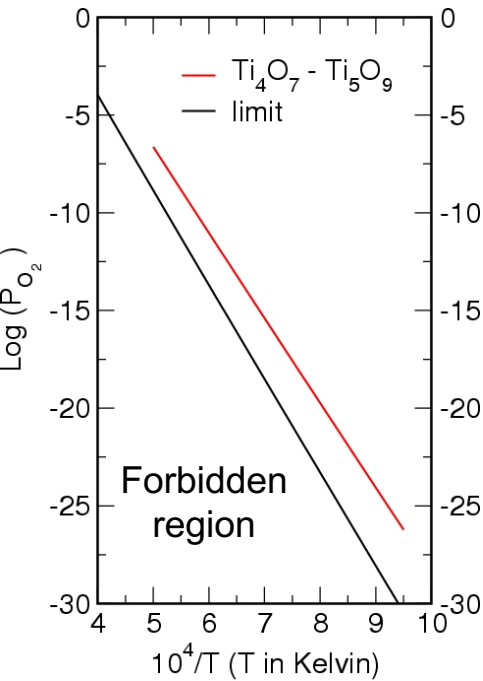


Figure 10b

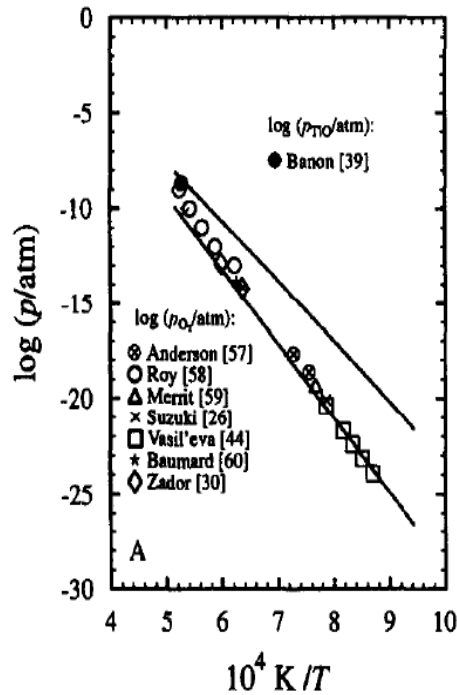


Figure 10c

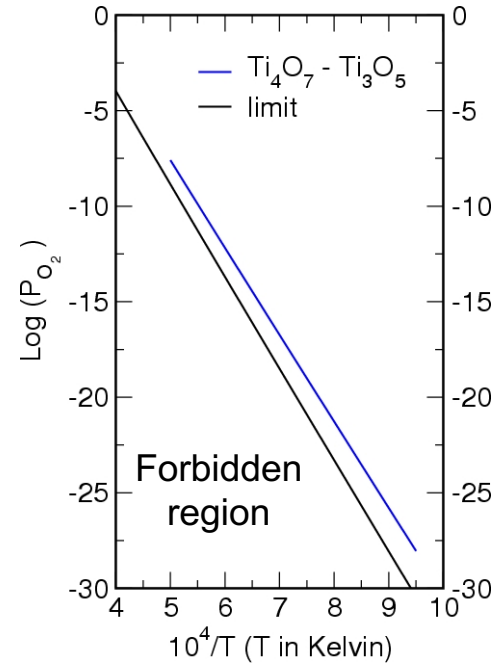
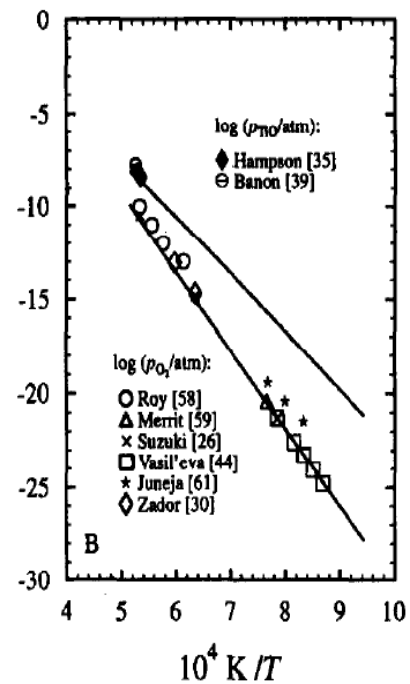


Figure 10d



# CRYSTAL Results for the Magneli phases

Figure 12a

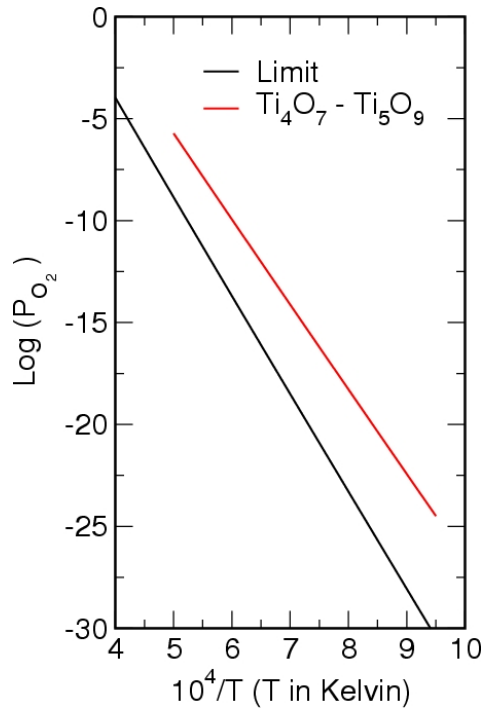


Figure 12b

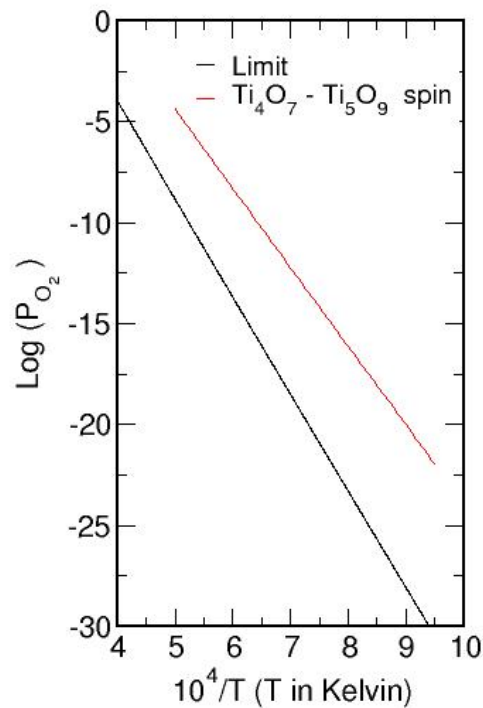
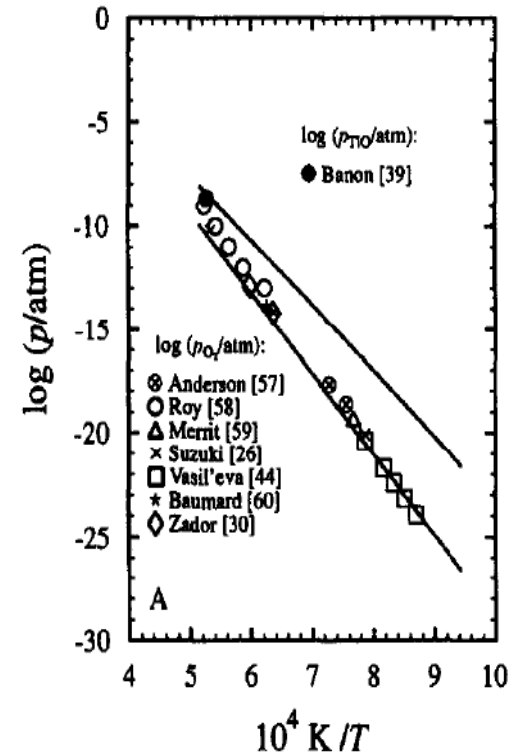
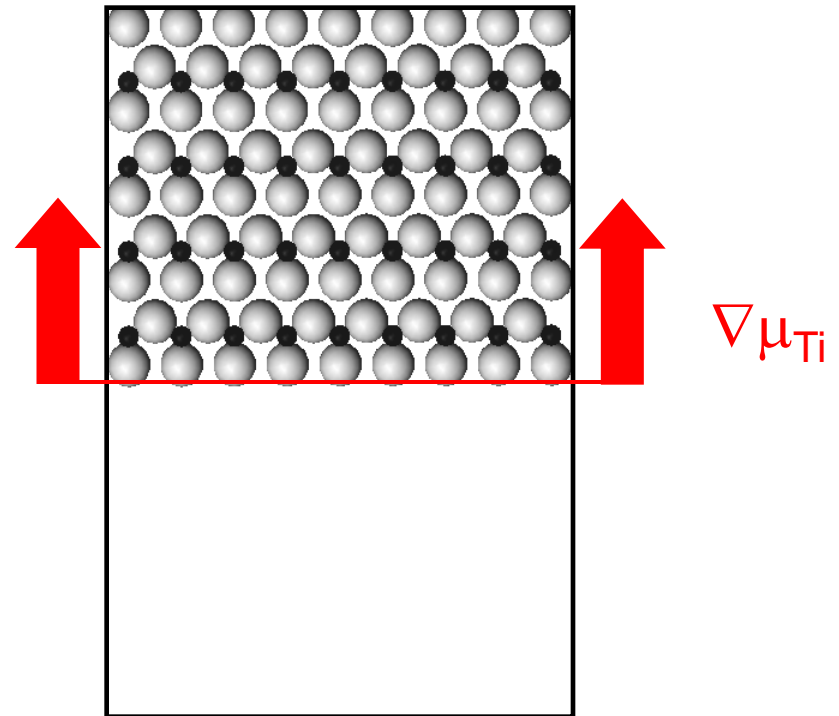
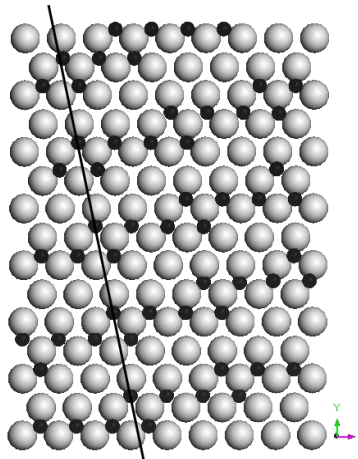
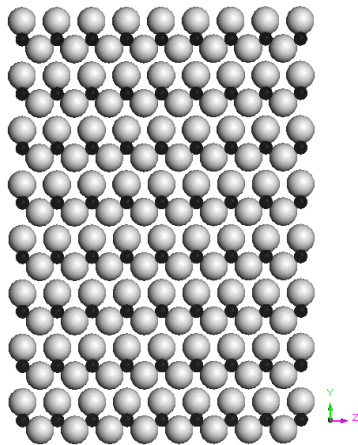


Figure 12c



# Formation mechanism for an oxygen-defective plane

Cation + anion (100) layer

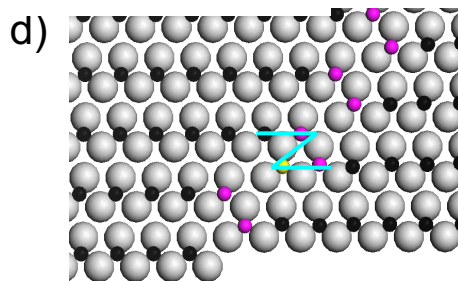
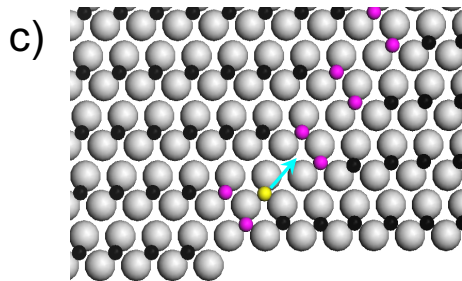
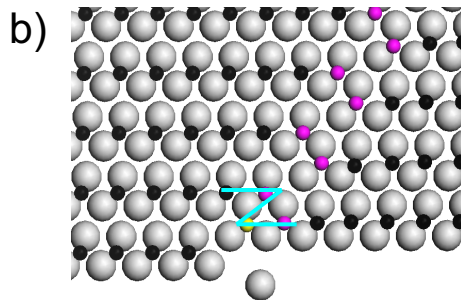
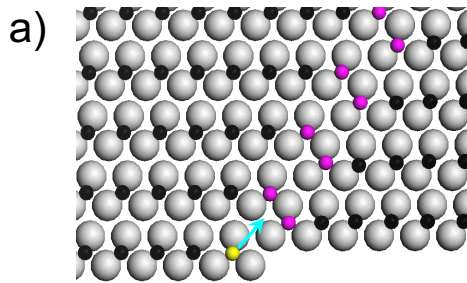


$$\mu_{\text{TiO}_2}^{\text{bulk}} = \mu_{\text{Ti}}^{\text{supercell}} + 2\mu_{\text{O}}^{\text{ref}}(p_{\text{O}_2}, T)$$

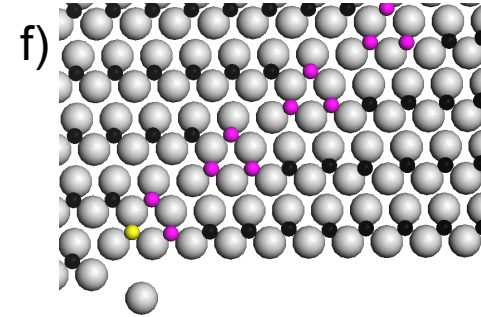
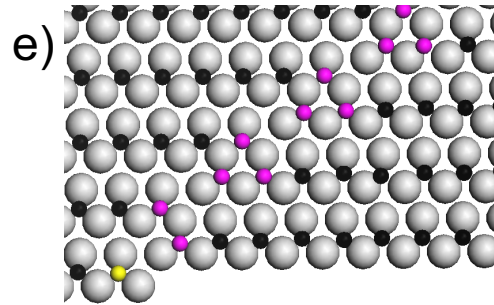
L. Bursill and B. Hyde, Prog. Sol. State Chem. Vol. 7, pp. 177, 1972.

S. Andersson and A. D. Waldsey, Nature Vol. 211, pp. 581, 1966.

# Formation mechanism for an oxygen-defective plane



Final stages



- Antiphase boundaries (dislocation) acts as high conductivity paths for titanium.
- Dislocations are needed
- No long-range diffusion
- Formation of Ti interstitials.

# Conclusions

- The thermodynamics of rutile's higher oxides has been investigated by first principles calculations.
- First principles thermodynamics reproduce the experimental observations reasonably well.
- Spin does not affect the thermodynamics.
- At a high concentration of oxygen defects and low oxygen chemical potential, oxygen defects prefer to form Magneli phases.
- But, at low concentration of oxygen defects and low oxygen chemical potential, titanium interstitials proved to be the stable point defects.
- These results support the mechanism proposed by Andersson and Waldsey for the production the crystalline shear planes in rutile.

# Acknowledgements

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- Dr. Barbara Montanari
  - Dr Keith Refson